EXAM IMAGE PROCESSING

30-1-2014

A FORMULA SHEET IS INCLUDED ON PAGES 3-4

Put your name on all pages which you hand in, and number them. Write the total number of pages you hand in on the first page. Write clearly and not with pencil or red pen. ***Always motivate your answers.*** *Success!*

Problem 1 Connected components (20 pt)
Consider a binary image f with pixel values 1 (foreground) and 0 (background) on a uniform pixel grid.

a. (5pt) Define the concept of *m-connected component* of the foreground, where $m = 4$ or $m = 8$; same for the background.

b. (10pt) Consider the binary image in Figure 1(a). Black dots represent foreground pixels, empty cells background pixels. Sketch the connected components of this image when choosing m-connectivity ($m = 4$ or $m = 8$) for the foreground and n-connectivity ($n = 4$ or $n = 8$) for the background, for all four combinations of m and n, i.e., (1) $m = 4, n = 4$; (2) $m = 4, n = 8$; (3) $m = 8, n = 4$; (4) $m = 8, n = 8$. Use upper case letters to label the pixels of the connected components of the foreground, and lower case letters for those of the background.

c. (5pt) Assume that the image in Figure 1(a) was obtained after digitizing a continuous image of a black ellipse on a white background. Which of the four combinations in the previous question would best represent the topology of the continuous image? Motivate your answer.

		•	•		
	•			•	
		•	•		

(a)

0	0	0	0	0
0	4	5	6	0
0	3	8	1	0
0	2	7	9	0
0	0	0	0	0

(b)

Figure 1: *(a): binary image. (b): grey scale image.*

Problem 2 Spatial filtering (25 pt)
Consider spatial filtering with a 3×3 mask of a grey scale image.

a. (3pt) Give the filter mask in the case of uniform filtering. What is the goal of uniform filtering?

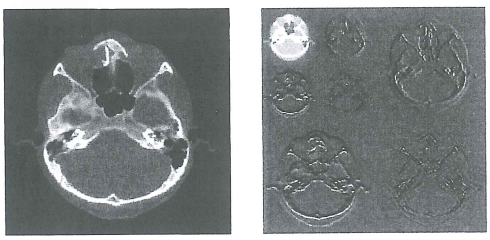
b. (3pt) Give the filter mask in the case of Laplacian filtering (several answers possible). What is the goal of Laplacian filtering?

c. (4pt) Define the median filter with a 3×3 mask and sketch the output image when applying this filter to the image in Figure 1(b). What is the goal of median filtering?

d. (5pt) Linear and space-invariant (LSI) image filters can be represented as a convolution with a kernel h called the "point spread function". Explain this name for h.

e. (10pt) Describe the main steps in the implementation of convolution filtering by means of frequency domain techniques.

8	4	6	2
6	2	4	0
4	8	8	4
2	6	6	2



(a) (b) (c)

Figure 2: *(a): grey scale image. (b): CT image. (c): two-level wavelet decomposition of (b).*

(continue on page 2)

Problem 3 Wavelet transforms (20 pt)

a. (5pt) Consider the following input vector: $c_0 = (8, 4, 6, 2)$. When the *unnormalized* 1-D Haar scaling and wavelet functions $h_\phi = \frac{1}{2}(1, 1)$, $h_\psi = \frac{1}{2}(1, -1)$ are used, the one-level ($J = 1$) wavelet decomposition of c_0 is the vector $(6, 4, 2, 2)$. Explain this result.

b. (5pt) Describe in words how you can build a J-level 2D discrete wavelet transform of an image by using separable scaling and wavelet functions. Explain the various subimages in the right image of Figure 2.

c. (5pt) Consider the 4×4 grey scale image in Figure 2(a). Compute the one-level ($J = 1$) 2D discrete wavelet transform of this image, assuming separable scaling and wavelet functions constructed from the unnormalized 1-D Haar scaling and wavelet functions, as defined above in question **a**.

d. (5pt) Explain how wavelet transforms can be used for image compression.

Problem 4 Morphological operations (15 pt)

Consider a digital binary image X consisting of N (filled) black squares on a white background, where the i^{th} square has a size of L_i by L_i pixels, with $L_i \geq 1$ an integer for all $i = 1, 2, \ldots, N$. The squares may touch or partly overlap; see Figure 3.

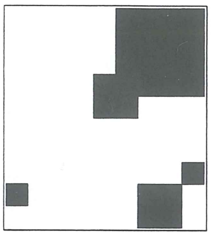


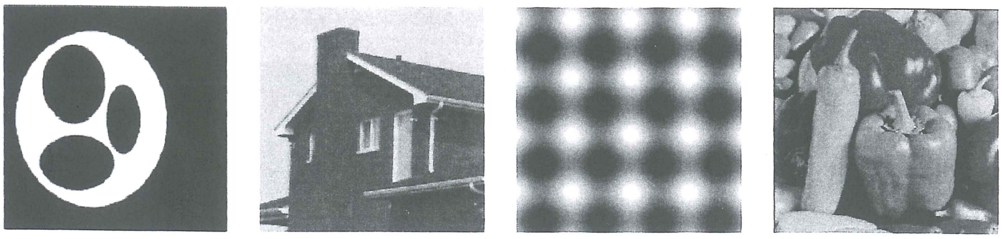
Figure 3: *Binary image X (the bounding box is not part of the image X).*

Assume that the number of squares and their sizes are unknown. Define a sequence of image operations on the image X, only making use of (i) set operations, and (ii) morphological operations, which produces the required result in the following cases:

a. (10pt) The output image should only contain the square(s) in the input image whose length/width is *at least* equal to a given integer value L (with $L \geq 1$).

b. (5pt) The output image should only contain the square(s) in the input image whose length/width is *precisely* equal to a given value L (with $L \geq 1$).

Problem 5 Image segmentation and description (20 pt)



(a) (b) (c) (d)

a. (10pt) Consider object segmentation. You can choose from the following collection of segmentation methods: (i) thresholding; (ii) Marr-Hildreth's method; (iii) split and merge; (iv) watershed method. For each of the four images in the figure above, indicate which segmentation method(s) you think perform(s) best, and why.

b. (10pt) Describe at least two contour descriptors and two regional descriptors which can characterize objects in binary or grey scale images.

Formula sheet

Co-occurrence matrix $g(i,j) = \{$no. of pixel pairs with grey levels (z_i, z_j) satisfying predicate $Q\}$, $1 \leq i,j \leq L$

Convolution, 2-D discrete $(f \star h)(x,y) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} f(m,n)\, h(x-m, y-n)$, for $x = 0,1,2,\dots,M-1$, $y = 0,1,2,\dots,N-1$

Convolution Theorem, 2-D discrete $\mathcal{F}\{f \star h\}(u,v) = F(u,v)\, H(u,v)$

Distance measures Euclidean: $D_e(p,q) = \sqrt{(p_1-q_1)^2 + (p_2-q_2)^2}$, City-block: $D_4(p,q) = |p_1 - q_1| + |p_2 - q_2|$, Chessboard: $D_8(p,q) = \max(|p_1-q_1|, |p_2-q_2|)$

Entropy, source $H = -\sum_{j=1}^{J} P(a_j) \log P(a_j)$

Entropy, estimated for L-level image: $\tilde{H} = -\sum_{k=0}^{L-1} p_r(r_k) \log_2 p_r(r_k)$

Error, root-mean square $e_{\mathrm{rms}} = \left[\frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \left(\widehat{f}(x,y) - f(x,y)\right)^2\right]^{\frac{1}{2}}$

Exponentials $e^{ix} = \cos x + i \sin x$; $\cos x = (e^{ix} + e^{-ix})/2$; $\sin x = (e^{ix} - e^{-ix})/2i$

Filter, inverse $\widehat{\mathbf{f}} = \mathbf{f} + \mathbf{H}^{-1}\mathbf{n}$, $\widehat{F}(u,v) = F(u,v) + \frac{N(u,v)}{H(u,v)}$

Filter, parametric Wiener $\widehat{\mathbf{f}} = \left(\mathbf{H}^t\mathbf{H} + K\,\mathbf{I}\right)^{-1} \mathbf{H}^t\mathbf{g}$, $\widehat{F}(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2 + K}\right] G(u,v)$

Fourier series of signal with period T: $f(t) = \sum_{n=-\infty}^{\infty} c_n\, e^{i\frac{2\pi n}{T}t}$, with Fourier coefficients: $c_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t)\, e^{-i\frac{2\pi n}{T}t}\, dt$, $\quad n = 0, \pm 1, \pm 2, \dots$

Fourier transform 1-D (continuous) $F(\mu) = \int_{-\infty}^{\infty} f(t)\, e^{-i2\pi\mu t}\, dt$

Fourier transform 1-D, inverse (continuous) $f(t) = \int_{-\infty}^{\infty} F(\mu)\, e^{i2\pi\mu t}\, d\mu$

Fourier Transform, 2-D Discrete $F(u,v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y)\, e^{-i2\pi(u\,x/M + v\,y/N)}$ for $u = 0,1,2,\dots,M-1$, $v = 0,1,2,\dots,N-1$

Fourier Transform, 2-D Inverse Discrete $f(x,y) = \frac{1}{MN} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u,v)\, e^{i2\pi(u\,x/M + v\,y/N)}$ for $x = 0,1,2,\dots,M-1$, $y = 0,1,\dots,N-1$

Fourier spectrum Fourier transform of $f(x,y)$: $F(u,v) = R(u,v) + i\,I(u,v)$, Fourier spectrum: $|F(u,v)| = \sqrt{R^2(u,v) + I^2(u,v)}$, phase angle: $\phi(u,v) = \arctan\left(\frac{I(u,v)}{R(u,v)}\right)$

Gaussian function mean μ, variance σ^2: $G_\sigma(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-(x-\mu)^2/2\sigma^2}$

Gradient $\nabla f(x,y) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right)$

Histogram $h(m) = \#\{(x,y) \in D : f(x,y) = m\}$. Cumulative histogram: $\mathsf{P}(\ell) = \sum_{m=0}^{\ell} h(m)$

Impulse, discrete $\delta(0) = 1$, $\delta(x) = 0$ for $x \in \mathbb{N}\backslash\{0\}$

Impulse, continuous $\delta(0) = \infty$, $\delta(x) = 0$ for $x \neq 0$, with $\int_{-\infty}^{\infty} f(t)\, \delta(t - t_0)\, dt = f(t_0)$

Impulse train $s_{\Delta T}(t) = \sum_{n=-\infty}^{\infty} \delta(t - n\Delta T)$, with Fourier transform $S(\mu) = \frac{1}{\Delta T} \sum_{n=-\infty}^{\infty} \delta(\mu - \frac{n}{\Delta T})$

Laplacian $\nabla^2 f(x,y) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$

Laplacian-of-Gaussian $\nabla^2 G_\sigma(x,y) = -\frac{2}{\pi\sigma^4}\left(1 - \frac{r^2}{2\sigma^2}\right) e^{-r^2/2\sigma^2}$ $\quad (r^2 = x^2 + y^2)$

Median The median of an odd number of numerical values is found by arranging all the numbers from lowest value to highest value and picking the middle one.

Morphology

Dilation $\delta_A(X) = X \oplus A = \bigcup_{a\in A} X_a = \bigcup_{x\in X} A_x = \{h \in E : \check{A}_h \cap X \neq \emptyset\}$,
where $X_h = \{x + h : x \in X\}$, $h \in E$ and $\check{A} = \{-a : a \in A\}$

Erosion $\varepsilon_A(X) = X \ominus A = \bigcap_{a\in A} X_{-a} = \{h \in E : A_h \subseteq X\}$

Opening $\gamma_A(X) = X \circ A := (X \ominus A) \oplus A = \delta_A \varepsilon_A(X)$

Closing $\phi_A(X) = X \bullet A := (X \oplus A) \ominus A = \varepsilon_A \delta_A(X)$

Hit-or-miss transform $X \circledast (B_1, B_2) = (X \ominus B_1) \cap (X^c \ominus B_2)$

Thinning $X \otimes B = X \backslash (X \circledast B)$, **Thickening** $X \odot B = X \cup (X \circledast B)$

Morphological reconstruction Marker F, mask G, structuring element B:
$X_0 = F$, $X_k = (X_{k-1} \oplus B) \cap G, \quad k = 1, 2, 3, \ldots$

Morphological skeleton Image X, structuring element B: $SK(X) = \bigcup_{n=0}^{N} S_n(X)$,
$S_n(X) = X \underset{n}{\ominus} B \setminus (X \underset{n}{\ominus} B) \circ B$, where $X \underset{0}{\ominus} B = X$ and N is the largest integer such that $S_N(X) \neq \emptyset$

Grey value dilation $(f \oplus b)(x, y) = \max\limits_{(s,t)\in B} [f(x - s, y - t) + b(s, t)]$

Grey value erosion $(f \ominus b)(x, y) = \min\limits_{(s,t)\in B} [f(x + s, y + t) - b(s, t)]$

Grey value opening $f \circ b = (f \ominus b) \oplus b$

Grey value closing $f \bullet b = (f \oplus b) \ominus b$

Morphological gradient $g = (f \oplus b) - (f \ominus b)$

Top-hat filter $T_{\text{hat}} = f - (f \circ b)$, **Bottom-hat filter** $B_{\text{hat}} = (f \bullet b) - f$

Sampling of continuous function $f(t)$: $\tilde{f}(t) = f(t)\, s_{\Delta T}(t) = \sum_{n=-\infty}^{\infty} f(t)\, \delta(t - n\Delta T)$.
Fourier transform of sampled function: $\tilde{F}(\mu) = \frac{1}{\Delta T} \sum_{n=-\infty}^{\infty} F(\mu - \frac{n}{\Delta T})$

Sampling theorem Signal $f(t)$, bandwidth $\mu_{\max}$: If $\frac{1}{\Delta T} \geq 2\mu_{\max}$, $f(t) = \sum_{n=-\infty}^{\infty} f(n\Delta T) \text{ sinc} \left[\frac{t - n\Delta T}{n\Delta T}\right]$.

Sampling: downsampling by a factor of 2: $\downarrow_2 (a_0, a_1, a_2, \ldots, a_{2N-1}) = (a_0, a_2, a_4, \ldots, a_{2N-2})$

Sampling: upsampling by a factor of 2: $\uparrow_2 (a_0, a_1, a_2, \ldots, a_{N-1}) = (a_0, 0, a_1, 0, a_2, 0, \ldots, a_{N-1}, 0)$

Set, circularity ratio $R_c = \frac{4\pi A}{P^2}$ of set with area A, perimeter P

Set, diameter $\text{Diam}(B) = \max\limits_{i,j} [D(p_i, p_j)]$ with p_i, p_j on the boundary B and D a distance measure

Sinc function $\text{sinc}\,(x) = \frac{\sin(\pi x)}{\pi x}$ when $x \neq 0$, and $\text{sinc}\,(0) = 1$. If $f(t) = A$ for $-W/2 \leq t \leq W/2$ and zero elswhere (block signal), then its Fourier transform is $F(\mu) = A\,W \text{sinc}\,(\mu W)$

Spatial moments of an $M \times N$ image $f(x, y)$: $m_{pq} = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} x^p\, y^q\, f(x, y), \quad p, q = 0, 1, 2, \ldots$

Statistical moments of distribution $p(i)$: $\mu_n = \sum_{i=0}^{L-1} (i - m)^n\, p(i)$, $m = \sum_{i=0}^{L-1} i\, p(i)$

Signal-to-noise ratio, mean-square $\text{SNR}_{\text{rms}} = \frac{\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \hat{f}(x,y)^2}{\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \left(\hat{f}(x,y) - f(x,y)\right)^2}$

Wavelet decomposition with scaling function h_ϕ, wavelet function h_ψ. For $j = 1, \ldots, J$:
Approximation: $c_j = \mathbf{H}c_{j-1} = \downarrow_2 (h_\phi * c_{j-1})$; Detail: $d_j = \mathbf{G}c_{j-1} = \downarrow_2 (h_\psi * c_{j-1})$

Wavelet reconstruction with dual scaling function $\tilde{h}_\phi$, dual wavelet function $\tilde{h}_\psi$. For $j = J, J-1, \ldots, 1$:
$c_{j-1} = \tilde{h}_\phi * (\uparrow_2 c_j) + \tilde{h}_\psi * (\uparrow_2 d_j)$

Wavelet, Haar basis $h_\phi = \frac{1}{\sqrt{2}}(1, 1)$, $h_\psi = \frac{1}{\sqrt{2}}(1, -1)$, $\tilde{h}_\phi = \frac{1}{\sqrt{2}}(1, 1)$, $\tilde{h}_\psi = \frac{1}{\sqrt{2}}(1, -1)$